Radial Electromagnetic Energy Flow and Plasma Heating Power for Theta Pinch Discharges

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In order to estimate radial electromagnetic energy flow directed toward a tube axis and plasma heating power for theta pinch discharges, experimental measurements with probes were performed. Time development for the spatial distributions of the electromagnetic energy flow and the plasma heating power density was obtained, so that some interesting phenomena of the theta pinch plasmas were revealed; that is, at the compression phase, the radial distribution of the Poynting vector has a hump under special discharge conditions and total plasma heating power shows a maximum at earlier time than the total electromagnetic input power does.

§1. Introduction

All the energy to produce a theta pinch plasma is supplied inductively only from the surface of the plasma through a cylindrical theta pinch coil. The energy is converted into the Joule heating energy, shock heating energy and electromagnetic field energy inside the coil and so on. The radially flowing energy of the theta pinch plasma is generally composed of electromagnetic energy flow, particle energy flow (energy flow with the moving particle), thermal conduction energy flow and radiative energy flow. If one can estimate the electromagnetic energy flow from measurements, the complicated behavior of the theta pinch plasma may be revealed from a new point of view.

Though numerous works have been published on the measurements of the magnetic field strength for various pinch plasmas, most of interests have been concentrated on the structure and the dynamics of current sheets.1 4) In this paper, we obtained the spatial distributions of the electromagnetic energy flow from the measurements for both the axial magnetic field $B_z$ and the azimuthal electric field strength $E_\theta$. These results made it possible to give more detailed explanation about the complicated phenomena of the theta pinch plasma than when one considered only the dynamics of the current sheets. Further, from the results of experimental measurements we calculated the total electromagnetic power supplied from the theta pinch coil, the plasma heating power and the time rate of the magnetic field energy, so that the characteristics of the heating for the theta pinch plasma were clarified by using time development of these power.

§2. Supplied Electromagnetic Energy and Plasma Heating Energy

Figure 1 is a schematic of a theta pinch plasma. Axial magnetic flux density $B_z$ is produced by coil current $I_c$ and modified by azimuthal current in the plasma. Introducing magnetic flux inside a theta pinch coil of a radius $R$

$$\Phi = \int_0^R B_z 2\pi r \, dr, \quad (1)$$

one can obtain the following equation for the coil voltage $V_c$ from Faraday's law:

$$V_c = -\frac{d\Phi}{dt} = 2\pi R E_\theta(R). \quad (2)$$

Here, $E_\theta(R)$ is azimuthal electric field at $r = R$. Total electric power supplied to the theta pinch coil, $P_e$, is given by an equation $P_e =$
If it is assumed that the theta pinch coil is an infinitely long solenoid, we can find the magnetic field between the coil and the discharge tube, \( H_z(R) \), by using Ampere's law:

\[ H_z(R) = \mu_0 I_c / l \]

where \( I_c \) is the axial length of the theta pinch coil. Therefore, one can obtain the following relationship for the electric power,

\[ P_e = V J_e = 2\pi R E_\theta(R) H_z(R) l = 2\pi R S_0 l = P_1(R) \]

where \( S_0 = E_\theta(R) H_z(R) \) is the value of the Poynting vector at \( r = R \) and \( P_1(R) = 2\pi R S_0 \) is the electromagnetic power supplied to the plasma column per unit axial length.

An energy-conservation theorem for electromagnetic field is given by the following equation

\[ \nabla \cdot S = \frac{\partial}{\partial t} \left( \frac{E \cdot D}{2} + \frac{H \cdot B}{2} \right) + E \cdot J, \quad (4) \]

where \( S = E \times H \) is the Poynting vector and other quantities are used in usual meaning. Equation (4) says that the electromagnetic energy flowing into unit plasma volume per unit time equals the time rate of the work done on the plasma by the fields \( E \cdot J \) plus the time rate of the electromagnetic field energy in the volume.

If one integrates eq. (4) over the inside of the cylinder of radius \( r \) (the plasma volume \( V \)), the energy-conservation equation per unit axial length of the cylindrical plasma becomes

\[ - \int_A S \cdot dA = \int_V \frac{\partial}{\partial t} \left( \frac{E \cdot D}{2} + \frac{B^2}{2\mu_0} \right) dv + \int_V E \cdot J \ dv, \quad (5) \]

where the left hand side of eq. (5) is an integration over the whole surface \( (A) \) of the plasma cylinder. In this experiment, the electric field energy \( D \cdot E/2 \) can be neglected as compared with the magnetic energy \( B^2/2\mu_0 \).

When it can be assumed that the plasma column is symmetric with respect to the axis and, in addition, has uniform property along the axis, eq. (5) is

\[ P_s(r) = P_0(r) + P_1(r), \quad (6) \]

where,

\[ P_s(r) = 2\pi r S(r), \quad (7) \]

\[ P_0(r) = \frac{1}{\mu_0} \int_0^r \left( \frac{\partial B_z}{\partial t} \right) B_z 2\pi a \ da, \quad (8) \]

\[ P_1(r) = \int_0^r E_\theta J_\theta 2\pi a \ da. \quad (9) \]

Here, \( S(r) \) is the radial component of \( S \) at radius \( r, J_\theta \) is the azimuthal current density and \( a \) is the variable. The electric power supplied from the plasma surface at radius \( r, P_1(r) \), is changed into the plasma heating power \( P_1(r) \) and the rate of magnetic field energy \( P_0(r) \).

If one compares eq. (3) with eq. (6) at the plasma surface \( (r = r_0) \) it is obvious that energy input by the external circuit is balanced by the sum of \( P_1(r_0) \) and \( P_0(r_0) \) when the loss of the electromagnetic power between the theta pinch coil and the tube can be neglected.

§3. Experimental Procedure

The theta pinch apparatus used in this experiment was powered by a 6 \( \mu \)F capacitor bank switched by an air spark gap. The discharge chamber consists of a 50 cm length of 6.2 cm i.d. Pyrex pipe. It was filled with argon gas after evacuated to the base pressure of about 10\(^{-5}\) Torr. The theta pinch coil which encircles the discharge tube has an inner diameter of 8 cm and a length of 20 cm. The device was operated under the conditions of applied voltages from 20 kV to 25 kV and filling pressures from 11 mTorr to 125 mTorr. The maximum value of \( I_c \) at 25 kV was 70 kA and its quarter period was about 3 \( \mu s \). \( I_c \) was measured by a Rogowski coil and a crowbar switch was not used in this experiment. Preionization was done by using a dc steady-state discharge between a hollow cathode and a cylindrical anode at an end of the discharge tube so that the shot to shot reliability of the discharges was satisfactory.

The axial magnetic field \( B_z \) was measured by means of a magnetic probe constructed of 100 turns of copper wire wound on a 1 mm nylon core. The coil was mounted axially in a 4 mm o.d. Pyrex tube and its tube was supported at an end of the discharge tube and moved radially. Radial profiles of time-integrated probe signals were obtained from every 2 mm radial scan at the midplane of the theta pinch coil.

Figure 2 is the typical profiles of \( I_c \) and \( B_z \) for a 25 mTorr plasma at the bank voltage of
25 kV. The magnetic probe was set at \( r=0 \). There is the abrupt rising of the magnetic field at the second half cycle (\( t=6.4 \) \( \mu \)s \( \sim \) 9.2 \( \mu \)s) and the third half cycle (\( t=12.4 \) \( \mu \)s \( \sim \) 15.6 \( \mu \)s) of the coil current \( I_c \). Therefore we can estimate that some effects of pinch phenomena appear at these half cycles.

The azimuthal component of electric field strength at radius \( r \), \( E_\theta(r) \), is obtained from Faraday’s law

\[
E_\theta(r) = -\frac{1}{r} \int_0^r \left( \frac{\partial B_z}{\partial t} \right) r \, dr.
\]

Thus, in addition to measurements of \( B_z \), \( E_\theta(r) \) was obtained separately by measuring the direct signal of the magnetic probe, \( \partial B_z/\partial t \), as a function of radius and by performing the integral with respect to \( r \). We chose a most reliable oscillogram from among several shots for measurements of \( B_z \) and \( \partial B_z/\partial t \) at each position of radius.

Moreover, we measured both the magnetic field between the theta pinch coil and the discharge tube and the total magnetic flux \( \Phi \) in order to calculate from eq. (3) the total electric power to the plasma column. A single turn loop encircled the middle of the discharge tube and a magnetic probe was set up in the space between the tube and the coil.

§4. Experimental Results and Discussions

4.1 Magnetic field and electric field

Typical distributions of \( B_z \) and \( E_\theta \) for a 25 mTorr plasma are shown in Figs. 3 and 4, respectively. The bank voltage was 25 kV. A point for maximum current density moves toward the axis with time. It is shown, for example, that radii for the point are 16 mm at \( t=8.0 \) \( \mu \)s and 8.5 mm at \( t=8.4 \) \( \mu \)s and average velocity of 1.9 cm/\( \mu \)s is obtained for the current sheet during the interval. From these profiles we can also estimate rough values of electrical conductivity and plasma temperature in the current sheet. It has been reported that the theoretical sheet thickness is equal to 5/2 \( \mu_0 \sigma u \), where \( \sigma \) is the mean value of the electrical conductivity in the sheet and \( u \) is the mean velocity. In Fig. 3 the thickness is about 1 cm at 8.4 \( \mu \)s, so that a conductivity of 1.05 \( \times \) 10\(^4\) \( \Omega \)/m can be obtained. If one uses the Spitzer’s relationship, this conductivity corresponds to a value of 4.8 \( \times \) 10\(^4\) K for the
plasma temperature. The electric field strength $E_0$ of Fig. 4 was calculated from eq. (10) by using measured values of $\partial B_2/\partial t$. In the case with no plasma the value of $\partial B_2/\partial t$ is uniform over the cross section of the plasma column. Since, then, a relationship $E_0(r)=-(1/2\pi)\partial(B_2/\partial t)$ can be derived from eq. (10), $E_0(r)$ increases linearly with radius. The profiles for $t=6.0 \mu s$ and $t=6.4 \mu s$ are near the case where plasma is not produced.

4.2 Radial electromagnetic energy flow and plasma heating power density

The Poynting vector $S(r)$ was obtained from Figs. 3 and 4 and the results are shown in Fig. 5. Using these profiles one can estimate the behavior of the pinch phenomena from a new point of view about electromagnetic energy transfer. On the other hand, the profiles of the magnetic field, which have been used so far, explain only the movement of the current sheets. It is found that there are special points at which electromagnetic energy does not flow radially. The points appear at $r=16.4$ mm when $t=7.6 \mu s$ and at $r=10.0$ mm when $t=8.0 \mu s$. It is shown that the electromagnetic energy flows inward at the outside region of the zero point of the Poynting vector but flows outward at the reversed field region, where the Poynting vector has a large positive power density near the plasma surface. After these times, however, the point corresponding to the maximum moves radially inward with time; i.e. the radii of the point are 16.5 mm at 8.0 $\mu s$ and 8.0 mm at 8.4 $\mu s$. High power density near the plasma surface at 7.2 $\mu s$ and 7.6 $\mu s$ is an evidence for the strong skin effect of high frequency discharge.

Another interesting behavior is that a hump appears on the profiles at $t=8.8 \mu s$. This shows that there is a marked region where the radial inward flux of the electromagnetic energy is larger at smaller radius. The hump of the Poynting vector is closely related to the $E_0$ profile at 8.8 $\mu s$ in Fig. 4. It is noticed from eq. (4) that at the maximum point plasma heating power equals the time rate of the magnetic field energy because $dS/dr$ becomes zero. Although an ideal method is to discuss the plasma behavior by measuring the total energy flowing radially, in this work only the electromagnetic energy flow was treated.

The calculation of plasma heating power density from measured values of $B_z$ and $E_0$ is given in Fig. 6 for two cases of pinch compression phase. The current density $J_\theta$ was calculated by using the equation $J_\theta=-(1/\mu_0)(\partial B_z/\partial r)$. For $t=7.2 \mu s$ and 7.6 $\mu s$, the heating power density $E_0 J_\theta$ has a maximum near the plasma surface. After these times, however, the point corresponding to the maximum moves radially inward with time; i.e. the radii of the point are 16.5 mm at 8.0 $\mu s$ and 8.0 mm at 8.4 $\mu s$. High power density near the plasma surface at 7.2 $\mu s$ and 7.6 $\mu s$ is an evidence for the strong skin effect of high frequency discharge.

Fig. 5. Some profiles of the Poynting vector $S(r)$ for a 25 mTorr plasma. The bank voltage is 25 kV.

Fig. 6. Profiles of plasma heating power density $E_0 J_\theta(r)$ for a 25 mTorr plasma operated at 25 kV. Fig. 6(a) and Fig. 6(b) correspond to the compression phases of the second half cycle and the third half cycle, respectively.
4.3 Time development of plasma heating power

Figure 7 is time variation of \( P_s = P_s(r_0), P_B = P_B(r_0) \) and \( P_J = P_J(r_0) \) which were obtained from eqs. (7), (8) and (9), respectively. The plasma radius of 28 mm was chosen because of limitation of magnetic measurements in this experiment. Experimental conditions of the bank voltage and the filling pressure were the same as above. It is shown that \( P_s, P_J \) and \( P_B \) have maxima at different times, respectively; at the compression phase of the second half cycle \((t=6.4 \mu s \sim 9.2 \mu s)\) the maximum occurs at 8.0 \( \mu s \) for \( P_s \), at 7.6 \( \mu s \) for \( P_J \) and at 8.4 \( \mu s \) for \( P_B \). However, for the compression phase of the third half cycle \((t=12.4 \mu s \sim 15.6 \mu s)\), only \( P_J \) has a maximum at a little earlier time than \( P_s \) and \( P_B \) do.

Here, we can examine the difference of times corresponding to maxima of \( P_s, P_J \) and \( P_B \). Fig. 8 shows the time variation of \( B_z \) and \( E_\theta \). The solid lines are the results measured by a magnetic probe and a magnetic loop set up outside the discharge tube \((R=36 \text{ mm})\). On the other hand, as to the dashed lines, the \( B_z \) profile (a) was measured at \( r_0=28 \text{ mm} \) and the \( E_\theta \) profile (b) at the same position was calculated from eq. (10) by obtaining the spatial distribution of \( \partial B_z / \partial t \). It is found that a maximum of \( P_s \) at the compression phase occurs near the time when \( E_\theta \) has a maximum and does not at the time when \( B_z \) has a maximum. It is also noticed from Fig. 6 that a peak of \( P_J \) at 7.6 \( \mu s \) results from the plasma heating by the current flowing the wide annular plasma with large radius. Then, the heating power for \( t=8.0 \mu s \) and \( t=8.4 \mu s \) is almost due to plasma current flowing in the current sheet which is moving radially inward.

If one compares the value of \( P_s \) with that of \( P_J + P_B \), the maximum difference of \( 2.0 \times 10^7 \text{ W/m} \) appears at 7.6 \( \mu s \). This value is 18\% of \( P_J + P_s \). Since in the case with no plasma the difference between the measured magnetic
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Figure 7 is time variation of $P_s=P_s(r_0)$, $P_B=P_B(r_0)$ and $P_1=P_1(r_0)$ which were obtained from eqs. (7), (8) and (9), respectively. The plasma radius of 28 mm was chosen because of limitation of magnetic measurements in this experiment. Experimental conditions of the bank voltage and the filling pressure were the same as above. It is shown that $P_s$, $P_1$ and $P_B$ have maxima at different times, respectively; at the compression phase of the second half cycle ($t=6.4 \mu s \sim 9.2 \mu s$) the maximum occurs at 8.0 $\mu s$ for $P_s$, at 7.6 $\mu s$ for $P_1$ and at 8.4 $\mu s$ for $P_B$. However, for the compression phase of the third half cycle ($t=12.4 \mu s \sim 15.6 \mu s$), only $P_1$ has a maximum at a little earlier time than $P_s$ and $P_B$ do.

Here, we can examine the difference of times corresponding to maxima of $P_s$, $P_1$ and $P_B$. Fig. 8 shows the time variation of $B_z$ and $E_0$. The solid lines are the results measured by a magnetic probe and a magnetic loop set up outside the discharge tube ($R=36 \text{ mm}$). On the other hand, as to the dashed lines, the $B_z$ profile (a) was measured at $r_0=28 \text{ mm}$ and the $E_0$ profile (b) at the same position was calculated from eq. (10) by obtaining the spatial distribution of $\partial B_z/\partial t$. It is found that a maximum of $P_s$ at the compression phase occurs near the time when $E_0$ has a maximum and does not at the time when $B_z$ has a maximum. It is also noticed from Fig. 6 that a peak of $P_1$ at 7.6 $\mu s$ results from the plasma heating by the current flowing the wide annular plasma with large radius. Then, the heating power for $t=8.0 \mu s$ and $t=8.4 \mu s$ is almost due to plasma current flowing in the current sheet which is moving radially inward.

If one compares the value of $P_s$ with that of $P_1 + P_B$, the maximum difference of $2.0 \times 10^7$ W/m appears at 7.6 $\mu s$. This value is 18% of $P_1 + P_s$. Since in the case with no plasma the difference between the measured magnetic
field and the analytical solution based on the electromagnetics was at least 15 per cent, the difference is considered to arise mainly from error due to measurements of $B_z$ and $\partial B_z/\partial t$ and destruction of plasma symmetry with respect to the tube axis.

We measured the time rate of total magnetic flux by means of a single turn magnetic loop and magnetic field in the space between the tube and the coil. Using these values the electromagnetic power supplied to the plasma column per unit axial length, $P_s(R)$, was calculated. The result is added to Fig. 7. It is shown that a great quantity of the electromagnetic power is lost at the region between radius $R$ and radius $r_0$; for example, the ratio of $P_s(r_0)$ to $P_s(R)$ is 0.41 at 7.6 $\mu$s and 0.37 at 8.0 $\mu$s. The greater part of the difference energy is considered to be used for the magnetic field energy in the space between the coil and the tube and the plasma heating energy dissipated just inside the tube wall.

4.4 Comparison with a previous experiment

Here, the method proposed in this paper was applied to an experimental result published previously. Benford$^8$ measured the spatial distributions of both $B_z$ and $E_\theta$ for a nitrogen plasma (the initial filling pressure 50 mTorr). In his experiment the theta pinch plasmas were produced by a 15 $\mu$F, 20 kV capacitor bank. The results are shown in Fig. 9. The dashed line is the profile for the Poynting vector calculated from these measured values of $B_z$ and $E_\theta$. It is noticed that a maximum for the value of the Poynting vector appears at $r=28$ mm. This hump is the same phenomena as that described in Fig. 5. The electron density and temperature were also measured by using a ruby laser beam. It was shown, as a result of the measurement for the above plasma, that the maximum electron temperature of $7 \times 10^4$ K occurred at $r=24$ mm and the maximum electron density of $6 \times 10^{16}$ cm$^{-3}$ at $r=26$ mm.

One can conclude from the above discussions about the experimental results given by Benford that the electromagnetic energy flux has a maximum at larger radius than the electron temperature and density have.

§ 4. Conclusion

As to the radial energy flow in the theta pinch plasma, the electromagnetic energy flow was investigated in detail. Consequently, we obtained a new result that the radial distribution of the Poynting vector has a hump under special discharge conditions. This phenomena depends on mainly the spatial distribution of $E_\theta$. Moreover, electromagnetic power to the plasma column and plasma heating power were obtained from the measurements. According to the consideration of the time variation of these power, it was practically found that in this experiment the plasma heating power showed a maximum at earlier time than the electromagnetic input power did.

Axial energy loss from the open ends has been a crucial problem for the linear theta pinch plasmas. Therefore, consideration of the axial electromagnetic energy flow may be possible if the method presented here is applied.

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References

4) M. Alidieires, R. Aymar, F. Koechlin, P. Jourdan.

